Phys 410
Spring 2013
Lecture #27 Summary
1 April, 2013

We considered Rutherford scattering and calculated the differential scattering cross section for scattering of an alpha particle (charge q) from a Au nucleus (charge Q). The two particles interact by means of the Coulomb force, which is parameterized as $F = \gamma/r^2$, with $\gamma = qQ/4\pi\varepsilon_0$. The orbit is a hyperbola, characteristic of an orbit of two particles interacting by means of a central inverse-square-law force with energy $E > 0$. By calculating the change in momentum of the alpha particle $|\Delta\vec{p}|$ in two ways, we found the relationship between the impact parameter and the scattering angle: $b = \frac{\gamma}{mv^2}\cot\theta/2$, where m is the alpha particle mass and v is its initial speed of the alpha when far from the nucleus. Putting this into the formula for the differential scattering cross section, $\frac{d\sigma}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$, we find: $\frac{d\sigma}{d\Omega} = \left(\frac{qQ/4\pi\varepsilon_0}{4E\sin^2(\theta/2)}\right)^2$.

This result was tested experimentally by Geiger and Marsden, who showed that the scattering rate scaled with n_{target} (by varying the thickness of the foil), scaled as $1/E^2$ (by varying the energy of the incident alpha particles) , scaled as $\frac{1}{\sin^4(\theta/2)}$ (by measuring the number of particles scattered vs. outgoing angle), and scaled as Z^2, where Q=+Ze is the nuclear charge.

Note that because $\frac{d\sigma}{d\Omega} \sim q^2Q^2$, the scattered particle distribution is insensitive to whether the Coulomb interaction is attractive or repulsive. Also, the agreement for the angular dependence of $\frac{d\sigma}{d\Omega}$ with data suggests that the Coulomb force has the simple $1/r^2$ dependence even down to nuclear length scales. Finally, the total scattering cross section calculated from this $\frac{d\sigma}{d\Omega}$ diverges. This is because the bare Coulomb force is infinitely long ranged. In reality, the Coulomb force of the nucleus is screened out by the electron cloud of the atom, on the length scale of one nm, or less. When this screening is taken into account the total scattering cross section becomes finite, as observed.

These calculations assume that the alpha particle only undergoes one scattering event in the material. In addition, because of the electron screening, when an alpha particle is near one nucleus, it is insensitive to all the other nuclei because they are 'cloaked' by their neutralizing electron clouds.